PRL-TH-94/08
March 1994

Degenerate Neutrinos in Left Right Symmetric Theory

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Abstract

Various hints on the neutrino masses namely, (*i*) the solar neutrino deficit (*ii*) the atmospheric neutrino deficit (*iii*) the need for the dark matter and/or (*iv*) the non-zero neutrinoless double beta decay collectively imply that all the three neutrinos must be nearlty degenerate. This feature can be understood in the left right symmetric theory. We present a model based on the group $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(2)_H$ which can explain the required departures from degeneracy in neutrino masses and large mixing among them without assuming any of the mixing in the quark or charged lepton sector to be large as would be expected in a typical $SO(10)$ model.

No laboratory experiment has unambiguously detected the mass for the neutrino so far. But there exists variety of hints [1] which when taken in totality [2, 3] are strong enough to suggest a definite pattern for the masses of the known neutrinos. These hints come from (*i*) deficit in the solar neutrinos [4] (*ii*) deficit in the low energy atmospheric neutrinos [5] (*iii*) need for about 30% hot dark matter[6] and (*iv*) indications that neutrinoless double beta decay may actually be taking place [7]. These observations when attributed to neutrino masses put strong restrictions on the neutrino (mass)^2 differences Δ and mixing $\sin^2 2\theta$ as well as the absolute values of their masses [1]. It was shown in ref. [2, 3] that any of the observation (*iii*) or (*iv*) when combined with (*i*) and (*ii*) imply that all the neutrinos must be nearly degenerate in mass if there are only three light neutrinos.

The near degeneracy of the neutrino masses is very different from the hierarchy observed in the masses of other fermions. But this seemingly different pattern can be naturally incorporated [2, 3] into the seesaw mechanism for the neutrino mass generation. This framework (when suitably augmented by a horizontal symmetry) is capable of explaining not only the near degeneracy in the neutrino masses but it can also lead [3] to the observed departures from the degeneracy. Specifically, one expects [3]

$$\begin{aligned} m_{\nu_1} &= m_0 - m_u^2/M, \\ m_{\nu_2} &= m_0 - m_c^2/M, \\ m_{\nu_3} &= m_0 - m_t^2/M \end{aligned} \tag{1}$$

where m_0 is the universal mass for the light neutrino while M represents the large majorana mass for the right handed neutrinos. The above equation leads to

$$\frac{|\Delta_{21}|}{|\Delta_{32}|} \approx \left(\frac{m_c}{m_t}\right)^2 \approx (1-3) \times 10^{-4} \tag{2}$$

This nicely reproduces the hierarchy required to simultaneously solve the solar and atmospheric neutrino problem. Moreover, if m_0 is $\sim$2 eV as required for the hot dark matter [6] or for obtaining the neutrinoless double beta decay [1] at the present experimental level, and if M is identified with the grand unification scale ($\sim 10^{16}$ GeV), one obtains [3] Δ_{12} in the range required for the solution of the solar neutrino problem through the MSW [9] mechanism.

The neutrino sector seems to be distinguished from other fermions also in respect of mixing among them. For example, the atmospheric neutrino problem [1] can be solved only if ν_μ is strongly mixed with the other neutrino say ν_τ. Typically, $\sin^2\ 2\theta_{\mu\tau} \sim .5$. The solution of the solar neutrino problem through the vacuum oscillations also needs large mixing between ν_e and say ν_μ. This is quite different from the quark sector where all the mixing angles are known to be small. One

would like to understand this feature of the neutrino sector along with their almost degeneracy. We present a possible way to understand this in the context of the left right symmetric theory and discuss it through an explicit model. [1]

The structure of the neutrino masses given in eq.(1) can arise form the following seesaw mass matrix:

$$M_\nu = \begin{pmatrix} m_0\ I & M_{\nu D} \\ M_{\nu D}^T & M\ I \end{pmatrix} \tag{3}$$

The above structure arises naturally [8] in a left right symmetric theory with an extra discrete parity D which connects the left and the right-handed sectors. Because of this symmetry, the breakdown of the $SU(2)_R$ at a high scale naturally induces a small vacuum expectation value (vev) for the left handed triplet Higgs field and thus leads to a non-vanishing contribution to the majorana masses of the left handed neutrinos. These dominate [8] over the conventional seesaw contribution for natural values of parameters. Hence, if some horizontal symmetry makes these masses identical, the physical neutrino masses would be almost degenerate. The conventional seesaw contribution then would lead to departures form this degeneracy with the structure similar to the one displayed in eq.(1).

While the basic scenario outlined above follows [2, 3] from simple considerations, the details require the presence of a complicated underlying structure specifically in the Higgs sector. A candidate model was first proposed in ref. [3] which used the horizontal $SU(2)$ symmetry. The present one as well as some of the recent models [11, 12] are also based on this symmetry. Our basic aim here is to understand both the departures of neutrino masses from degeneracy as well difference in mixing pattern between neutrinos and quarks in a qualitatively different manner compared to ones presented in [3, 11, 12].

In the context of a typical seesaw model, one expects [14] relations not only between the masses of neutrinos and other fermions but also between their mixings. As a result, the large mixing is not natural in this case. There one expects $M_{\nu D}$ in eq.(3) to be similar to a typical up quark mass matrix and M_{down} to $M_{leptons}$. The mixing in the neutrino sector is then related to the quark sector and hence would be expected to be small. This can be avoided if the majorana mass matrix for the right handed neutrinos has some texture [15]. This possibility does not exist in the present case because of the assumed left right symmetry. This would automatically make the right handed majorana mass matrix proportional to the left handed majorana mass matrix. The latter is required to be proportional to identity (see eq.(3)) if one wants to have degenerate masses for the neutrinos. We propose a way out which

[1] Various models have been recently proposed [10, 11, 12, 13] while the present work was in the progress. We shall make a comparison of these models with the present one at the end.

naturally leads to differences in the mixing pattern between quarks and neutrinos without giving up the basic left right symmetry.

We retain the underlying left right symmetric framework which naturally explains the degeneracy and work for simplicity with the gauge group $G_{LR} \equiv SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. An $SU(2)_H$ symmetry is introduced to obtain the degenerate neutrinos and a $G_{LR} \times SU(2)_H$ singlet fermion N is introduced to obtain different mixing pattern among neutrinos compared to other fermions. The $SU(2)_H$ group can either be a softly broken global symmetry or could be gauged. Fermionic generations are taken to transform as triplets under $SU(2)_H$ and the Higgs fields transform as follows under $SU(2)_L \times SU(2)_R \times SU(2)_H$:

$$\begin{array}{cc} \Phi_{ab} \sim (2,2,5) & \Phi \sim (2,2,1) \\ \Delta_L \sim (3,1,1) & \Delta_R \sim (1,3,1) \\ \phi_L \sim (2,1,3) & \phi_R \sim (1,2,3). \end{array} \tag{4}$$

The Yukawa couplings of the quarks ($Q_{L,R}$) and leptons ($l_{L,R}$) are given by

$$\begin{aligned} \mathcal{L}_Y \;=\; & h_q\, \overline{Q}_{aL} \Phi Q_{aR} + h'_q\, \overline{Q}_{aL} \hat{\Phi} Q_{aR} + \gamma_q\, \overline{Q}_{aL} \Phi_{ab} Q_{bR} + \gamma'_q\, \overline{Q}_{aL} \hat{\Phi}_{ab} Q_{bR} \\ & + h_l\, \overline{l}_{aL} \Phi l_{aR} + h'_l\, \overline{l}_{aL} \hat{\Phi} l_{aR} + \gamma_l\, \overline{l}_{aL} \Phi_{ab} l_{bR} + \gamma'_l\, \overline{l}_{aL} \hat{\Phi}_{ab} l_{bR} \\ & + f \left[l^T_{aL} C \epsilon \tau . \mathbf{\Delta_L} l_{aL} + L \leftrightarrow R \right] + h_N \left[\overline{l}_{aL} \phi_{aL} + L \leftrightarrow R \right] \; N + \frac{1}{2} M \; N^T \; C N \end{aligned} \tag{5}$$

Where $a, b = 1, 2, 3$ are the generation indices; $\hat{\Phi} \equiv \tau_2 \Phi^* \tau_2$. Both the Φ and Φ_{ab} contain two neutral fields which could acquire vacuum expectation value (vev). We shall denote by $\kappa, \kappa' (\kappa_{ab}, \kappa'_{ab})$ the vev of the neutral components contained in $\Phi(\Phi_{ab})$. The mass matrices for the charged fermions ($f = U, D, E$) are then given by

$$\begin{aligned} (M_U)_{ab} &= (h_q \kappa + h'_q\, \kappa') \delta_{ab} + \gamma_q\, \kappa_{ab} + \gamma'_q\, \kappa'_{ab} \\ (M_D)_{ab} &= (h_q \kappa' + h'_q\, \kappa) \delta_{ab} + \gamma_q\, \kappa'_{ab} + \gamma'_q\, \kappa_{ab} \\ (M_E)_{ab} &= (h_l \kappa' + h'_l\, \kappa) \delta_{ab} + \gamma_l\, \kappa'_{ab} + \gamma'_l\, \kappa_{ab} \end{aligned} \tag{6}$$

The structure of the neutrino masses is more complicated. Assume that the $SU(2)_R$ symmetry is broken by the large vev of Δ_R and ϕ_R. The vev of the right handed triplet automatically induces the vev for the left handed triplet [8] if the potential is to respect $G_{LR} \times D$. This follows from the following types of terms in the Higgs potential

$$V_\Delta = \mu^2\; Tr(\Delta^\dagger_L \Delta_L + \Delta^\dagger_R \Delta_R) + \lambda\; Tr\left((\Delta^\dagger_L \Delta_L)^2 + (\Delta^\dagger_R \Delta_R)^2 \right) + \delta\; Tr\left(\Delta^\dagger_L \Phi \Delta_R \Phi^\dagger \right) + ... \tag{7}$$

Where we have retained only typical terms which lead to the following relation at the minimum (more general analysis can be found in [8])

$$< \Delta_L >< \Delta_R > \approx \gamma\kappa^2 \tag{8}$$

where, γ is related to the parameters in V_Δ and κ is a typical $SU(2)_L$ breaking vev of the field Φ. Very similar hierarchy also exists among the vev of the fields $\phi_{L,R}$. This would follow form the terms in the Higgs potential of the following type:

$$V_\phi = \mu'^2(\phi_L^\dagger\phi_L + \phi_R^\dagger\phi_R) + \lambda'\left((\phi_L^\dagger\phi_L)^2 + (\phi_R^\dagger\phi_R)^2\right) + \delta'\left(\phi_L^\dagger\Phi\phi_R\right) + ... \tag{9}$$

This leads to the following relation

$$< \phi_L >< \phi_R > \approx \gamma'\kappa\delta' \tag{10}$$

Hence if the vev for ϕ_R is required to be very large as we will do in the following then the induced vev for ϕ_L will automatically be suppressed. This suppresses the mixing of the left handed neutrinos with the field N allowing at the same time a large mixing between the right handed neutrinos and N. If we neglect the former mixing then the neutrino mass matrix is given in the basis (ν_L^c, ν_R, N) by

$$\mathcal{M}_\nu = \begin{pmatrix} m_0\, I & \hat{M}_{\nu D} \\ \hat{M}_{\nu D}^T & \hat{M}_R \end{pmatrix} \tag{11}$$

We have the following form for various matrices:

$$\hat{M}_{\nu D} \approx \left(\begin{array}{c|c} & 0 \\ M_{\nu D} & 0 \\ & 0 \end{array} \right) \tag{12}$$

$$\hat{M}_R \approx \begin{pmatrix} M_0 & 0 & 0 & M_1 \\ 0 & M_0 & 0 & M_2 \\ 0 & 0 & M_0 & M_3 \\ M_1 & M_2 & M_3 & M \end{pmatrix} \tag{13}$$

$M_a \equiv h_N < \phi_{Ra} >$; $M_{\nu D}$ is a 3×3 matrix following from eq.(5):

$$(M_{\nu D})_{ab} = (h_l\kappa + h_l'\kappa')\delta_{ab} + \gamma_l\kappa_{ab} + \gamma_l'\kappa_{ab}' \tag{14}$$

It follows from eqs(5) and (8) that

$$M_0 = f < \Delta_R > \qquad m_0 = f^2\frac{\gamma\kappa^2}{M_0} \tag{15}$$

The effective masses of the three light neutrinos are given by the matrix:

$$m_{eff.} \approx m_0 I - M_{\nu D} M_R^{-1} M_{\nu D}^T \tag{16}$$

with

$$M_R^{-1} \approx \frac{1}{D}\left(\begin{array}{ccc} D_1 & M_1\,M_2\,M_0 & M_1\,M_3\,M_0 \\ M_1\,M_2\,M_0 & D_2 & M_2\,M_3\,M_0 \\ M_1\,M_3\,M_0 & M_2\,M_3\,M_0 & D_3 \end{array}\right) \tag{17}$$

where,

$$\begin{aligned} D_1 &= M_0(M_0M - M_2^2 - M_3^2) \\ D_2 &= M_0(M_0M - M_1^2 - M_3^2) \\ D_3 &= M_0(M_0M - M_2^2 - M_1^2) \\ D &= M_0^2(M_0M - M_2^2 - M_3^2 - M_1^2) \end{aligned} \tag{18}$$

It follows from eqs.(16,17) that the terms induced by the coupling between N and the right handed neutrinos allow for a general mixing among neutrinos even in the extreme case of the diagonal $M_{\nu D}$. This therefore allows us to understand the observed difference in the mixing among neutrinos compared to other fermions. While this possibility can be realized in general, in the following we discuss a specific case which has the merit of being economical. In this example, the mixing in the quark sector is correlated to that in the leptonic sector. Normally, such a situation would arise in typical models based on $SO(10)$. The additional $SU(2)_H$ symmetry turns out be restrictive in our case and a similar situation can be realized even with the gauge group G_{LR}. This happens if (i) all the primed Yukawa couplings in eq.(5) are set to zero. This would be true in the supersymmetric theory or if one imposes some softly broken Peccei Quinn symmetry [16] and (ii) The vev κ_{ab} and κ'_{ab} are real. The κ_{ab} can then be chosen diagonal by a proper $SU(2)_H$ rotation. In this case, eqs. (6,14) can be used to show that

(a) The $M_{\nu D}$ and M_U are diagonal.

(b) The following relations hold among various masses:

$$\frac{m_b - m_d}{m_\tau - m_e} = \frac{m_s - m_d}{m_\mu - m_e} \tag{19}$$

$$\frac{m_t - m_u}{m_c - m_u} = \frac{m_3 - m_1}{m_2 - m_1} \tag{20}$$

$m_{1,2,3}$ are the eigenvalues of $M_{\nu D}$. Eq.(19) relates the leptons and the quark masses and follows here without using any grand unification. This is is seen to be reasonably well satisfied. Note however that both these relations would be expected to receive corrections if the group $G_{LR} \times SU(2)_H$ is broken at a high scale.

(c) The matrices M_D and M_E can be diagonalized by the same matrix which would

coincide with the Kobayashi Maskawa matrix in this case. [2] Hence, the mixing among the charged leptons cannot be large and we shall neglect it completely in the following. The large mixing among neutrinos could come about because of the presence of the additional singlet which allows the M_R^{-1} to have a texture and a general form given by eq.(17). The details will depend upon various parameters entering the matrix $m_{eff.}$. We discuss below a specific choices for the ranges of parameters needed to obtain the realistic pattern. We will assume M_1 to be very small and set it to zero. The neutrino masses then are given by

$$m_{eff.} \approx m_0\ I - \frac{1}{D}\begin{pmatrix} m_1^2 D_1 & 0 & 0 \\ 0 & m_2^2 D_2 & M_0 M_2 M_3 m_2 m_3 \\ 0 & M_0 M_2 M_3 m_2 m_3 & m_3^2 D_3 \end{pmatrix} \tag{21}$$

The masses $m_{1,2,3}$ are restricted by eq.(20). This allows for a non-hierarchical values of $m_{1,2,3}$ but we assume more natural possibility of the hierarchy $m_1 \ << \ m_{2,3}$. In this case, eq.(20) implies

$$\frac{m_c}{m_t} \approx \frac{m_2}{m_3}. \tag{22}$$

Realistic pattern for the mixing and masses now require the hierarchy $M_0 \sim M \ < M_2 \ < M_3$. In this limit, the mixing between the second and the third neutrino and the masses of the neutrinos are given by

$$\begin{aligned} m_{\nu 1} &\approx m_0 - \frac{m_1^2}{M_0} \\ m_{\nu 2} &\approx m_0 - \frac{m_2^2\ M \cos^2\theta_{23}}{M_2^2} \\ m_{\nu 3} &\approx m_0 - \frac{m_2^2}{M_0 \sin^2\theta_{23}} \\ \tan^2\theta_{23} &\approx (\frac{m_c}{m_t})^2\ (\frac{M_3}{M_2})^2 \end{aligned} \tag{23}$$

It follows from there that

$$\frac{|\Delta_{32}|}{|\Delta_{21}|} \approx \frac{M_2^2}{M M_0 \tan^2\theta_{23}} \qquad \Delta_{21} \approx -2 m_0 m_2^2 \frac{M\ \cos^2\theta_{23}}{M_2^2} \tag{24}$$

Typically, for

$$\frac{M_2}{M_3} \sim \quad \frac{(M\ M_0)^{1/2}}{M_2} \sim \frac{1}{30} \tag{25}$$

[2] Note that with the assumption (i) and (ii) above, the Kobayashi Maskawa matrix becomes real even if h_q and γ_q are complex. Thus one has to look for CP violation elsewhere or has to relax these assumptions. We shall not discuss these issue here as our main motivation is the neutrino sector.

one obtains

$$
\begin{aligned}
M_0 &\approx 10^{13}\mathrm{GeV} \\
\sin^2 2\,\theta_{23} &\approx 0.5 \\
\frac{|\Delta_{21}|}{|\Delta_{32}|} &\approx (\frac{m_c}{m_t})^2 \sim 10^{-4}
\end{aligned}
\tag{26}
$$

if $\Delta_{21} \sim 10^{-6}\mathrm{eV}^2$ and $m_2 \sim m_c$. These values are in the right range needed to solve both the atmospheric neutrino and the solar neutrino problem. Moreover, with the M_0 given in eq.(26), the common mass m_0 is fixed to be in the eV range (see eq. (15)) as required for solving the dark matter problem if $f^2\gamma \sim 1$. It is seen from eq.(25) that if $M \sim M_0 \sim 10^{13}$ GeV then M_3 is fixed to be around the grand unification scale. Thus, one needs to associate, two physically distinct scales with the vev of Δ_R and ϕ_R. These different scales then lead to the hierarchy in the values of Δ_{21} and Δ_{23}. The $\nu_e - \nu_\mu$ mixing would depend on the KM matrix and on M_1 both of which we have ignored in the above for simplicity. It should be possible to generate the desired mixing when these parameters are kept non-zero in view of the general structure (eq. (17)) possessed by $m_{eff.}$

If we set $M_{1,2,3}$ zero in eq.(17) then, the additional singlet N decouples. This corresponds to the usual situation. In this case, one could understand the large mixing among neutrinos if either M_E is unrelated to M_D and thus could admit a large mixing or the Dirac masses for the neutrinos are unrelated to the up quark masses. While such relations are typical of $SO(10)$ and are interesting from the point of view of economy, they are not automatic and can be avoided in the $SO(10)$ model by invoking more Higgs fields. This has been used to generate large mixing in ref. [11, 12] where the presence of a doublet vev coming from the 126 dimensional representation lead to inequality of M_D and M_E. In this case, one could have a large mixing among charged leptons without conflicting with the small mixing of the KM matrix. Note that in our case also the M_D and M_E could be unrelated if the primed couplings in eq.(6) are retained. In such a case, one could accommodate the large mixing between neutrinos without invoking any singlet fermions. But the singlet fermion makes it possible to have large mixing even without sacrificing the relationship between $M_D(M_U)$ and $M_E(M_{\nu D})$.

The present model is a logical combination of the models in ref [3] and [10]. Bamert and Burgess used the singlet in order to generate departures from degenerate neutrino spectrum using the conventional seesaw mechanism. In the present case, the left right symmetry naturally explains the dominance of the degenerate mass term over the family dependent contributions. The singlet is used here mainly to decouple the mixing in the neutrino sector from the rest of fermions.

In summary, we have discussed a specific model to generate the almost degenerate spectrum for the neutrino masses and mixing. The model presented here provides a concrete example of the proposals in ref. [2, 3]. The salient feature of the model is the left right symmetry and a gauge singlet fermion which together is shown to lead to the pattern of neutrino masses and mixing desired from the point of view of solving the solar, atmospheric and the dark matter problem simultaneously. While we have worked here with the left right symmetric gauge group G_{LR}, the required scales call for embedding of this group into an $SO(10)$ type of grand unified model with an intermediate scale around 10^{13} GeV.

Acknowledgments

I would like to thank participants of the Workshop on High Energy Physics Phenomenology held at Madras, India- particularly- K.S. Babu, C. Burges S. Rindani and J. Valle for numerous discussions related to the present work and to J. Valle and C. Burges for informing me about their work.

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